Multiple Integration

"You cannot believe in GOD until you believe in yourself" ...Swami ji.

Before solving the questions, you must learn all the general formula for single integration. First think about the single integration then you can solve this double integration problem. Here, if you have some confusion about first limit x or y, this means you are thinking in right direction. So don't worry about dxdy or dydx, you can use anything dxdy or dydx in constant limit but you need to define first. Here I am using first limit with respect to first derivative in constant limit. If limit in variable, then you can also decide yourself (THINK). Whenever integration with respect to x then y as constant and vice-versa.

Solve:

Q.1 $\int_0^3 \int_0^2 xy\ (1+x+y)dxdy$ Ans. $30\frac{3}{4}, 39$

Q.2 $\int_1^2 \int_0^{3y} y\, dydx$ Ans. 7

Q.3 Evaluate $\iint xydxdy$ over the region in the positive quadrant for which x+y<1 Ans. ${}^{1}/_{24}$

Q.4 $\iint \sqrt{4x^2-y^2}dxdy = \frac{1}{3}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$

Q.5 $\iiint (x+y+z)dxdydz$ where the region bounded by the planes x=0, y=0, z=0 & x+y+z=a, (a>0) Ans. $\frac{a^4}{8}$

Q.6 $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3}dx$

Q.7 $\int_0^a \int_0^b (x^2+y^2)dxdy$ Ans. $\frac{1}{3}ab(a^2+b^2)$

Q.8 $\int_0^1 \int_0^2 (x+2)dxdy$ Ans.5

Q.9 $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$ Ans. $\frac{1}{x}\pi log(1+\sqrt{2})$

Q.10 $\int_1^a \int_1^b \frac{dxdy}{xy}$ Ans. log a log b

Q.11 $\int_1^2 \int_0^x \frac{dxdy}{x^2+y^2}$ Ans. $\frac{1}{4}\log_e 2$

Q.12 $\int_0^{\pi/2} \int_{\pi/2}^{\pi} cos(x+y)dxdy$ Ans. -2

Q.13 $\int_0^1 \int_0^{\sqrt{1-y^2}} ydydx$ Ans. ${}^{1}/_{3}$

Q.14 $\int_0^1 \int_0^{x^2} e^{y/x}dxdy$ Ans. ${}^{1}/_{2}$

Q.15 $\int_0^1 \int_0^{\sqrt{y}} (x^2+y^2)dxdy$ Ans. ${}^{3}/_{35}$

Q.16 $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\, dxdy$ Ans. $\pi\, {}^{a^3}/_{6}$

Q.17 $\int_0^2 \int_0^{\sqrt{2x-x^2}} xdxdy$ Ans. ${}^{\pi}/_{2}$

Q.18 $\int_0^a \int_0^{\sqrt{a^2-x^2}} (a^2-x^2-y^2)\ dxdy$ Ans. ${}^{\pi a^2}/_{8}$

Q.19 (i) $\int_0^a \int_0^{\sqrt{a^2+x^2}} (x+y)\, dxdy$ Ans. ${}^{2a^3}/_{3}$

(ii) $\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2\, ydxdy$ Ans. ${}^{a^5}/_{15}$

(iii) $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{dydx}{1+x^2+y^2}$ Ans. ${}^{\pi}/_{4}\log(1+\sqrt{2})$

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<u>"Arise! Awake! and stop not until the goal is reached"</u>

Q.20 (i) $\iint_A (x^2+y^2)\,dxdy$ Ans. $1/16$

(ii) $\iint_A (2x+3y)dxdy$ where A is the region bounded by x=0, y=0, x+y=1 Ans. $\frac{1}{3}\left(e^3-\frac{3}{2}e^2+\frac{1}{2}\right)$

Q.21 Evaluate $\iint x^2y^2\,dxdy$ over the region $x^2+y^2\le 1$, $\pi/24$

Q.22 $\iint xy\,(x+y)dxdy$ over the area between the parabola y=x^2 & the line y=x Ans. $3/56$

Q.23 $\iint \frac{x}{\sqrt{1-y^2}}dxdy$ over the positive quadrant of the circle $x^2+y^2=1$ Ans. $1/6$

Q.24 Evaluate $\iint x^{m-1}y^{n+1}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Ans.
$\frac{a^m b^n}{4}\,\frac{\sqrt{m/2}\sqrt{n/2}}{\sqrt{\frac{n}{2}+\frac{m}{2}+1}}$

Q.25 $\iiint_V \frac{dxdydz}{(x+y+Z+1)^3}$ where the region V is bounded by the planes x=0, y=0, z=0, x+y+z=1
$\frac{1}{2}\log_2 - 5/16$

Q.26 $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dxdydz$ Ans. $\frac{3}{8}log2-\frac{19}{9}$

Q.27 Evaluate $\iiint x^p y^q z^r\,dxdydz$, where x,y,z are positive & x+y+z=≤ 1 Ans. $\frac{\sqrt{p+1}\sqrt{q+1}\sqrt{r+1}}{\sqrt{p+q+r+4}}$

Q.28 $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dxdydz$ Ans. $\frac{8}{3}abc(a^2+b^2+c^2)$

Q.29 $\int_0^{\pi/2}\int_0^{a\cos\theta} r\sin\theta\,d\theta dr$ Ans. $a^2/6$

Q.30 $\int_0^{a}\int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+b^2}\,dxdy$ by changing it to coordinates & hence proofs that I=$\frac{1}{20}\pi a^5$

Q.31 Integrate $r^2\sin\theta$ over the area of the coordinate r=$a(1+\sin\theta)$ above the initial line
Ans.$\frac{16}{15}a^4$

Q.32 $\int_0^{\pi}\int_0^{a\sin\theta} rd\theta dr$ Ans. $\pi^2/4$

Q.33 $\int_0^{\pi}\int_0^{a(1+\cos\theta)} rd\theta dr$ Ans. $\frac{3}{4}\pi a^2$

Q.34 $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta\,d\theta dr$ Ans. $\frac{5}{8}\pi a^3$

Q.35 $\int_0^{\pi/2}\int_0^{2a\cos\theta} r^2\sin\theta\cos\theta d\theta dr$ Ans. $\frac{8}{15}a^2$

Q.36 Evaluate $\iint \frac{rd\theta dr}{\sqrt{a^2+r^2}}$ over one loop the lemnisccate $r^2=a^2\cos 2\theta$ Ans. $1/2\,(4-\pi)a$

Q.37 Evaluate $\iint r^2 d\theta dr$ over the area of the circle r=acos θ Ans. 4a$3/a$

Multiple Integration

“The world is the great gymnasium where we come to make ourselves strong.”

Evaluate the following double integrals by changing to polar coordinate

Q.38 $\int_0^1 \int_0^x \frac{x^3 dxdy}{\sqrt{x^2+y^2}}$ ans. $1/_4 \log(1+\sqrt[2]{2})$

Q.39 $\int_0^a \int_0^a \frac{xdydx}{x^2+y^2}$ Ans.$\pi\, a/_4$

Q.40 $\int_0^1 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dxdy$ Ans. $\frac{3x}{8}-1$

Q.41 $\int_0^2 \int_0^{2x-4} \frac{2y-1}{x+1} dxdy$ Ans. 36+42log 3

Q.42 $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}=\pi^2/_4$

Q.43 $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ hence show that $\int_0^\infty e \quad dx = 2\pi/_2$

Q.50 Change the order of integration in $\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x,y)dx\,dy$

Q.51 (i) Change the order of integration in $\int_0^{2a} \int_{\sqrt{6ax}}^{\sqrt{16a^2-x^2}} f(x,y)\,dx\,dy$

(ii) Change the order of integration in the following integral $\int_0^{2a\sqrt{3}} \int_{\sqrt{6ax}}^{\sqrt{16a^2-x^2}} f(x,y)\,dy\,dx$

Q.52 Prove that i=$\int_0^{\pi/2} \int_0^{\pi/2} \sin x \sin^{-1}(\sin x \sin y\, dx\, dy = \frac{\pi}{2}\left(\frac{\pi}{2}-1\right)$

Q.53 Show that $\int_0^a \int_0^{a-x} f(x,y)dxdy = \int_0^a \int_0^{a-y} f(x,y)dx\,dy$

Q.54 Evaluate the integral by changing the order of integration $\int_0^\infty \int_0^x xe^{-x^2/y}\,dxdy$

Q.55 Show that $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} f(x,y)dxdy$=$\int_0^a \int_0^{\sqrt{(a^2-y^2)}} f(x,y)dydx$

Q.56 Change the order of integration in the system of integrals

$\int_0^{\pi/2} \int_0^{a(1+\cos\theta)} f(r,\theta)r\,d\theta dr$+$\int_{\pi/2}^{\pi} \int_0^{a(a+\cos\theta)} f(r,\theta)\,r\, d\theta dr$

Q.57 Change the order of integration in $\int_0^{\pi/2} \int_0^{2a\cos\theta} f(r,\theta)d\theta dr$

Q.58 Prove that

(i) $\int_a^b \int_a^x f(x,y)dxdy = \int_a^b \int_y^b f(x,y)\,dydx$

(ii) $\int_0^a \int_0^{a-x} f(x,y)dxdy = \int_0^a \int_0^{a-y} f(x,y)dydx$

Q.59 Change of order of integration in $\int_0^a \int_0^{lx} V dxdy$, where V is a function of x and y

Q.60 Change the order of iteration in $\int_0^a \int_0^{\sqrt{(a^2-x^2)}} f(x,y)$dxdy

Q.61 Evaluate the following by changing the order of integration $\int_0^1 \int_{e^x}^e \frac{dxdy}{\log y}$

Q.62 Evaluate the integral by changing the order of integration in $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{xdxdy}{\sqrt{(x^2+y^2)}}$

Q.63 Change the order of integration in $\int_0^\infty \int_0^x xe^{-x^2} dxdy$ and hence evaluate it

Q.64 Prove that $\int_0^{2a} \int_0^{\sqrt{(2ax-x^2)}} f(x,y)dxdy = \int_0^a \int_{a-\sqrt{(a^2-y^2)}}^{a+\sqrt{(a^2-y^2)}} f(x,y)dydx$

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"God helps those who help themselves." …Swami ji

Q.65 Change the order of integration in $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}\frac{\varphi'(y)(x^2-y^2)xdxdy}{\sqrt{\{4a^2x^2-(x^2+y^2)^2\}}}$ and hence evaluate it.

Q.66 Change the order of the integration in $\int_0^a\int_{\sqrt{ax-x^2}}^{\sqrt{(ax)}} V dxdy$

Q.67 Change the order the integration in $\int_0^1\int_{\frac{1}{2}\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} V dxdy$ where V is a function of x and y

Q.68 Change the order of the integration in $\int_0^1\int_x^{x(2-x)} f(x,y)dxdy$

Q.69 Change the order of the integration in $\int_c^a\int_{(b/a)\sqrt{(a^2-x^2)}}^{b} V\ dxdy, c < a$, where V is a function of x and y

Q.70 Show that $\int_a^b dx\int_{a^2/x}^{x} f(x,y)dy = \int_{a^2/b}^{a} dy\int_{a^2/y}^{b} f(x,y)dx + \int_a^b dy\int_y^b f(x,y)dx$

Q.71 Change the order of integration in the following integral $\int_0^{\frac{1}{2}a}\int_{x^2/a}^{x-x^2/a} f(x,y)dxdy$

Q.72 Change the order of integration in $\int_0^a\int_0^x \frac{f'(y)dxdy}{\sqrt{\{(a-x)(x-y)\}}}$, and hence find its value.

Q.73 Show how the change in order of integration leads to the evaluation of $\int_0^\infty \frac{\sin rx}{x}dx$ from $\int_0^\infty\int_0^\infty e^{-xy}\sin rx\, dxdy$.

HINT:- Integrating with respect to y, the second integral reduce to the first. Now evaluate the second integral by changing the order of integration.

Q.74 If n ≥ 0, show that $\int_a^b\int_a^y (y-x)^n f(x)dydx = \frac{1}{n+1}\int_a^b (b-x)^{n+1}f(x)dx$.

HINT:- Change the order of integration

Q.75 Change the order of integration in $\int_0^{\pi/2}\int_0^{2a\cos\theta} f(r,\theta)r\, d\theta dr$

Q.76 Change the order of integration in $\int_0^{\pi/3}\int_{a sec^2\theta/2}^{(8a\cos\theta)/3} f(r,\theta)d\theta dr$

Dirichlet Integration and Lioville's theorem:

Q.77 The triple integral $\iiint_V x^{i-1}y^{n-1}z^{n-1}dxdydz$

Q.78 If V is the region given by x $x \geq 0, y \geq 0, z \geq 0$ and $x+y+z \leq 1$, then $\iiint_V x^{i-1}y^{m-1}z^{n-1}dxdydz = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+1)}$

Q.79 Liouville extended Dirichlet's theorem as fallows: If x,y,z are all positive such that $h_1 \leq x + y + z \leq h_2, then\ \iiint f(x+y+z)\, x^{l-1}y^{n-1}z^{n-1}dxdydz = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+)}\int_{h_1}^{h_2} f(u)\, u^{l+m+n-1}$du to the first order of approximation.

Q.80 Evaluate $\iiint_R dxdydz$, where R is the region bounded by the planes $x=0, y=0, z=0, x+y+z=6$.

Q.81 Evaluate $\iiint x^{l-1}y^{m-1}z^{n-1}\, dxdydz$, where x,y,z are always positive but limited by the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$.

Q.82 Evaluate $\iiint xyz\, dx\, dy\, dz$ taken throughout the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \leq 1$

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“When an idea exclusively occupies the mind, it is transformed into an actual physical or mental state.” .....Swami Vivekananda

Q.83 Evaluate the integral $\iiint e^{x+y+z}\, dx\, dy\, dz$ taken over the positive octant such that $x+y+z \leq 1$ with the help of Liouville’s theorem.

Q.84 Evaluate $\iiint_R (x+y+z+1)^2\, dx\, dy\, dz$, where R is the region $x \geq 0, y \geq 0, z \geq 0, x+y+z$ where R is the region $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1$.

Q.85 Evaluate the integral $\iiint \frac{dz\, dy\, dz}{1+(x+y+z)^3}$ taken over the positive octant such that $x+y+z \leq 1$.

Q.86 Find $\iiint x^{-1/2}\, y^{-1/2} z^{-1/2}(1-x-y-z)^{1/2}\, dxdydz$, where integral is taken over all positive values of the variable subject to the condition $x+y+z < 1$.

Q.87 Evaluate of the integral $\iiint \frac{dx\, dy\, dz}{\sqrt{(a^2-x^2-y^2-z^2)}}$, the integral being extended for all positive values of the variables for which the expression is real.

Q.88 Evaluate $\iiint \sqrt{\left(\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}\right)}\, dxdydz$, integral being taken over all positive value of x, y, z such that $x^2+y^2+z^2 \leq 1$.

Q.89 Evaluate $\iiint \sqrt{(a^2b^2c^2-b^2c^2x^2-c^2a^2y^2-a^2b^2z^2)}\, dx\, dy\, dz$ taken throughout the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

Q.90 Show that $\iiint x^{l-1}\, y^{n-1} z^{n-1} dx\, dy\, dz = a^l b^m c^n \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+1)} h^{l+m+n,}$ where the integral is extended over all non-negative values of x, y, z such that $x/a+y/b+z/c \leq h$.

Q.91 Evaluate the integral $\iiint \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right) xyz\, dx\, dy\, dz$ over the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, lying in the positive octant.

Q.92 (i) If l,m,n are all positive, show that $\iiint x^{l-1} y^{n-1}\, z^{n-1} dx\, dy\, dz = \frac{a^l b^m c^n.\Gamma(\frac{1}{2}l)\Gamma(\frac{1}{2}m)\Gamma(\frac{1}{2}n)}{8\Gamma\left\{\frac{1}{2}(l+m+n)+1\right\}}$, where the multiple integral is taken throughout the part of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, which lies in the positive octant.

(ii) Find the volume of the following ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

Hint: - Volume $=8\iiint dx\, dy\, dz = 8 \times \frac{1}{6}\pi abc$, where the integral is taken throughout the part of ellipsoid in the positive octant

Q.93 (i) Show that $\iiint \frac{dx\, dy\, dz}{(x+y+z+1)^3} = \frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$ throughout the volume bounded by the co-ordinate planes and the planes and the plane $x+y+z=1$.

(ii) Show that $\iiint \frac{dx\, dy\, dz}{(x+y+z+1)^3} = \frac{1}{16} log\left(\frac{256}{e^5}\right)$ taken throughout the tetrahedron bounded by the planes $x=0, y=0, z=0,\ x+y+z=1$.

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Multiple Integration

Q.94 Prove that $\int_0^\infty \int_0^\infty f(x+y)x^\alpha y^\beta dxdy = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} \int_0^\infty f(u)u^{\alpha+\beta+1}du$ when extended to all positive values of variable x and y, subject to the condition $x+y<\infty$.

Q.95 Find the value of $\iiint \log(x+y+z)\, dx\, dy\, dz$, the integral extending over all positive value of x, y, z subject to the condition $x+y+z<1$.

Q.96 (i) Prove that $\iiint \frac{dx\, dy\, dz}{\sqrt{(1^2-x^2-y^2-z^2}} = \frac{\pi^2}{8}$, the integral being extended to all positive values of the variables for which the expression real.

(ii) If S is the unit sphere wit its center at the origin, then prove that $\iiint_S \frac{dx\, dy\, dz}{s\sqrt{(1-x^2-y^2-z^2)}} = \pi^2$

Q.97 Prove that the area in the positive quadrant between the curve $x^n+y^n=a^n$ and the exes is $\frac{a^2[\Gamma(1/n)]^2}{2n\Gamma(2/n)}$.

Hint. Area $=\iint dx\, dy$, subject to the condition $0\le x^n+y^n\le a^n, i.e. (x/a)^n+(y/a)^n\le 1$.

Q.98 Evaluate $\iiint dx\, dy\, dz$, where the variables all positive and subject to the condition $x^n+y^n+z^n\le a^n$.

Q.99 Evaluate $\iint x^{1/2}y^{1/2}\,(1-x-y)^{2/3}\, dx\, dy$ over the domain D bounded by the lines $x=0, y=0$ and $x+y=1$.

Q.100 Prove that $\iint_D e^{-x^2-y^2} dx\, dy = \frac{\pi}{4}(1-e^{-R^2})$, where D is the region defined by $x\ge 0, y\ge 0, x^2+y^2\le R^2$.

Q.101 With certain limitations on the value of a,b,m and n prove that

$\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)}\, x^{2m-1}y^{2n-1}dx\, dy = \frac{\Gamma(m)\Gamma(n)}{4a^m b^n}$.

Q.102 Evaluate $\iiint_R (x+y+z+1)^2\, dx\, dy\, dz$, where R is the region defined by $x\ge 0, y\ge 0, z\ge 0, x+y+z\le 1$.

Q.103 Evaluate $\iint_R \sqrt{(}x^2+y^2)dx\, dy$, where R is the region in the first quadrant of xy –plan bounded by $x^2+y^2=4$ and $x^2+y^2=9$.

Q.104 Evaluate $\iiint_D (1-x-y-z)^{p-1}\, x^{l-1}y^{m-1}z^{n-1}dx\, dy\, dz$, where D is interior of the tetrahedron bounded by the planes $x=0, y=0, z=0, x+y+z=1$.

Q.105 Evaluate $\iiint x^l y^m\, z^n(1-ax-by-cz)^p dx\, dy\, dz$ over the volume of tetrahedron formed by $x=0, y=0, z=0, ax+by+cz=1$.

Q.106 $\int_0^1 \int_y^1 x^2\cos(x^2-xy)dy\, dx = \int_0^{asin\alpha} \int_0^{y\cot\alpha} x^2\cos(x^2-xy)dx\, xy = \frac{1}{2}(1-\cos 1)$

Q.107 $\int_0^{a\cos\alpha} \int_{xtan\alpha}^{\sqrt{a^2-x^2}} f(x,y)\, dx\, dy = \int_a^b \int_y^b f(x,y)dydx$.

Q.108 $\int_0^b \int_0^{2x} f(x,y)dx\, dy = \int_a^b \int_y^b f(x,y)dx\, dy$.

Q.109 $\int_0^a \int_0^{2x} f(x,y)dx\, dy = \int_0^\infty \int_{y/2}^\infty f(x,y)dy\, dx$

Q.110 $\int_0^4 \int_0^{2\sqrt{x}} f(x,y)dx\, dy = \int_0^4 \int_{y^2/4}^4 f(x,y)dy\, dx$.

Q.111 $\int_0^3 \int_1^{\sqrt{4-x}} (x+y)\, dy\, dx = \int_1^2 \int_0^{4-y^2} (x+y)dx\, dy$.

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Q.112 $\int_0^a \int_{mx}^{lx} v(x,y)\,dx\,dy = \int_{ma}^{la} \int_{y/l}^{a} v(x,y)dy\,dx + \int_0^{ma} \int_{y/l}^{y/m} v(x,y)dydx.$

Q.113 $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} v(x,y)dx\,dy = page\ \ 4/8\ \ G.C.\ Chaddha = I_1 + I_2 + I_3.$

Q.114 $\int_0^{2a} \int_{x^2/4a}^{3a-x} vdxdy = I_1 + I_2.$

Q.115 $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)dx\,dy = \int_0^a \int_0^{\sqrt{a^2y^2}} f(x,y)dy\,dx.$

Q.116 $\int_0^{\pi/2} \int_0^{2a\cos\theta} f(r_1\theta)d\theta dr = \int_0^{2a} \int_0^{cos^1 r/2} f(r_1\theta)drd\theta\,.$

Q.117 $\int_2^a \int_x^{a^2/x} f(x,y)dxdy.$

Q.118 $\int_0^a \int_{x^2/2}^{2a-x} f(x,y)dxdy = \int_0^a \int_0^{\sqrt{ay}} f(x,y)dydx + \int_a^{2a} \int_0^{2a-y} f(x,y)dydx.$

Q.119 $\int_0^\infty \int_x^\infty e^{-y}/y\,dydx = \int_0^\infty \int_0^y \frac{e^{-y}}{y}dydx = 1$

Q.120 $\int_0^1 \int_x^{\sqrt{2x^2}} \frac{xdxdy}{\sqrt{x^2+y^2}}$ Ans:- $1 - \frac{1}{2}\sqrt{2}$

Q.121 $\int_0^1 \int_{\sqrt{ax}}^{a} \frac{y^2}{\sqrt{y^4-a^2x^2}}dxdy$ Ans:- $\frac{\pi a^2}{c}$

Q.122 $\int_0^a \int_y^a e^{x^2}\,dydx$ Ans:- $\frac{1}{2}(e^{a^2} - 1)$

Q.123 If $x \geq 0$ prove that $\int_a^b \int_a^y (y-x)^x f(x)dydx = \frac{1}{x+1}\int_a^b (b-x)^{x+1} f(x)dx.$

Q.124 Show that $\int_0^x dx \int_0^{a-x} f(x,y)dy = \int_0^x dy \int_0^{a-y} f\,(x,y)dx.$

Q.125 Show that $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} vdxdy = \int_0^{4a} \int_{y^2/4a}^{2\sqrt{ay}} vdydx$

Q.126 Evaluate the following integral

A. $\int_0^a \int_0^b (x^2+y^2)dxdy$ Ans:- $\frac{ab}{3}(a^2+b^2)$ B. $\int_0^1 \int_0^{x^2} e^{y/x}dydx$ Ans:- 1

C. $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$ Ans:- $\frac{\pi}{4}\log(1+\sqrt{2})$

D. $\iint xydxdy$, over the region in the positive quadrant for which $x + y \leq 1$ Ans:- $1/24$

E. $\iint_R xydxdy$, where R $(x \geq 0, y \geq 0, x^2 + y^2 = a^2)$ Ans:- $a^2\pi/4$

F. $\iint_A xydxdy$, where A is the area bounded by x-axis, x=2a, and the curve $x^2 = 4ay$ Ans:- $a^4/3$

G. $\iint_A \sqrt{xy - y^2}dxdy$, where A is the triangle with vertices (0,0),(10,1), and (1,1)

H. $\iint_a dxdy$, where A is the region in the first quadrant bounded by the hyperbola $x\,y = 16$
And $y = x, y = 0$,& $x = 8$

I. $\iint (x^2+y^2)dxdy$, in the area enclose by the curves $y = 4x, x + y = 3, y = 0,$ & $y = 2$

Q.127 Evaluate $\int_0^2 \int_0^{\sqrt{2-x^2}} (x^2+y^2)dxdy$

Q.128 Evaluate $\iint\ \ r^2 drd\theta\ R$ (inside the circle r=a and out side the cardioids. $r = a(1-\cos\theta)$)

Q.129 Evaluate $\iint_A \frac{rdrd\theta}{\sqrt{r^2+a^2}}$ where A is a loop of $r^2 = a^2\cos 2\theta$

Q.130 Evaluate $\iint_A r^2 \sin\theta d\theta dr$ where A is a $r = 2a\ \cos\theta$ a above initial line.

Q.131 $\iint_R (x^2+y^2)dxdy$ where R is the region bounded by $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2 (a < b)$

Multiple Integration

Q.132 Express as a single integral and Evaluate

A. $\int_0^{\frac{a}{\sqrt{2}}}\int_0^x xdxdy + \int_{\frac{a}{\sqrt{2}}}^{a}\int_0^{\sqrt{a^2-x^2}} xdxdy$

B. $\int_0^1\int_0^y (x^2+y^2)dydx + \int_1^2\int_0^{2-y}(x^2+y^2)dydx$

Q.133 $\int_0^a\int_y^a \frac{xdxdy}{x^2+y^2}$ by changing to polar co-ordinates.

Q.134 Evaluate $\int_0^{\pi}\int_0^a r^2\sin\theta\cos\theta drd\theta$ by transforming in Cartesian co-ordinate system.

Q.135 Evaluate $\int_0^1\int_0^{1-x}\frac{ydxdy}{e^{x+y}}$ by using the transformation $x+y=u, y=uv$.

Q.136 Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\sqrt{\frac{\sin\emptyset}{\sin\theta}}\,d\emptyset d\theta$ by using the transformation $x=\sin\emptyset\cos\theta$, $y^{==}\sin\emptyset\sin\theta$

Q.137 Change the order of integration.

(a) $\int_0^1\int_{x^2}^{2-x} xydxdy$ & evaluate

(b) $\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} Vdxdy$ $Ans{:}-\int_0^a\int_{y^2/2a}^{a-\sqrt{a^2-y^2}} vdxdy + \int_0^a\int_{a+\sqrt{a^2-y^2}}^{2a} vdxdy + \int_a^{2a}\int_{y^2/2a}^{2a} vdxdy.$

(c) $\int_0^5\int_{2-x}^{2+x} f(x,y)dxdy$

(d) $\int_0^a\int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2+y^2}} xydxdy$ and evaluate

(e) $\int_0^a\int_0^x \frac{f'(y)dxdy}{\sqrt{(a-x)(x-y)}}$ and evaluate

(f) $\int_0^a\int_{\frac{1}{2}\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} Vdxdy$ Ans:- $\int_0^{a/2}\int_{\sqrt{a^2-4y^2}}^{\sqrt{a^2-y^2}} Vdxdy + \int_{a/2}^{a}\int_0^{\sqrt{a^2-y^2}} vdydx$

(g) $\int_0^{\frac{a}{2}}\int_{\frac{x^2}{a}}^{x-\frac{x^2}{a}} f(x,y)dxdy$ Ans:- $\int_0^{a^2/4}\int_{a/2-1/2\sqrt{a^2-4ay}}^{\sqrt{ay}} f(x,y)dydx.$

(h) $\int_0^{\infty}\int_0^{\infty} e^{-xy}\sin nx dxdy$ and Show that $\int_0^{\infty}\frac{\sin nx}{x}dx=\frac{\pi}{2}$

(i) $\int_0^1\int_0^y (x^2+y^2)dxdy + \int_1^2\int_0^{2-y}(x^2+y^2)dxdy$

(j) $\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} f(r,\theta)rdrd\theta$.

(k) $\int_0^{\frac{\pi}{3}}\int_{asec^2\frac{\theta}{2}}^{(8a\cos\theta)/3} f(r,\theta)drd\theta$

Q.138 Find the mass of a loop of the lemniscates $r^2=a^2\sin 2\theta$ if density is $\rho=kr^2$

Q.139 Find the area lying inside the cardioids $r=a(1+\cos\theta)$ and outside the circle $r=a$.

Q.140 Find the area lying inside the cardioids $y^2=4ax$ and $x^2=4ay$

Q.141 Find the area lying inside the cardioids $y=2-x$ and $y=2(2-x)$.

Q.142 By double integral find the volume of torus generated by the revolution of circle $x^2+y^2=4$ about line $x=3$.

Q.143 By double integral find the volume generated by the revolution of the cardioids $r=a(1+\csc\theta)$ about initial line.

Q.144 Find the area of the carve $a^2x^2=y^3(2a-y)$

Q.145 Find the area enclose by the cardioids $r=a(1+\cos\theta)$ & $r=a(1-\cos\theta)$

Q.146 Find the area include between the carves $y^2=4a(x+a)$ and y^2=$4b(b-x)$

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Multiple Integration

Q.147 Find the area common on the circle $x^2 + y^2 = a^2$ & $x^2 + y^2 = 2ax$

Q.148 Find the average density of the sphere of radius a whose density at a distance r from the center of sphere $\rho = \rho_0 \left[1 + k\frac{r^3}{a^3}\right]$

Q.149 Find the area included between the curves $y = x^2 - 6x + 3$ & $y = 2x + 9$

Q.150 Find the C.G. of the area of the corve $r = a(1 + \cos\theta)$.

Q.151 Find the C.G. of the area in the positive quadrant of the curve $x^{2/3}+y^{2/3}=a^{2/3}$

Q.152 Find the C.G. of the arc of the curve $x = a(t + \sin t)y = a(1 - \cos t)$

Q.153 Evaluate $\iiint \; xyz\, dxdydz$ where $s = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, \geq 0\}$

Q.154 $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{(a^2-r^2)}{a}} r dz dr d\theta$

Q.155 $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ taken throughout the volume of the sphere of unit radius, lying in the first octant.

Q.156 Change the order of integration in the system of the integrals $\int_0^{\pi/2} \int_0^{a(1+\cos\theta)} f(r,\theta) r d\theta dr + \int_{\pi/2}^{\pi} \int_0^{a(1+\cos\theta)} f(r,\theta) r d\theta dr$

Q.157 Transform the multiple integral $\iiint v dxdydz$ by the polar transformation $x = r\sin\theta \; \sin\emptyset$, $y = r\sin\theta\cos\emptyset$, $z = r\cos\theta$.

Q.158 Transform to polar co-ordinates and integrates $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+x^2}} dxdy$, the integral being extending over all positive value of x and y subject to $x^2 + y^2 \leq 1$.

Q.159 Transform the integral $\iiint V dxdydz$, when $x = r\sqrt{(1 - m^2 sin^2\emptyset)}$, and $y = r\sin\emptyset\sqrt{(1 - n^2 sin^2\theta)}$, $z = r\cos\theta\cos\emptyset$ and $m^2 + n^2 = 1$.

Q.160 Find the value of $\int_0^a \int_0^b \frac{dxdy}{(c^2+x^2+y^2)^{3/2}}$ by transforming it to polar.

Q.161 Show the transforming to polar co-ordinate that $\int_0^{a\tan\theta} \int_0^{a\tan\beta} \frac{dxdy}{(x^2+d^2+c^2)^2} = (1/2a^2)\{\sin\alpha tan^{-1}(\tan\beta\cos\alpha) + \sin\beta tan^{-1}(\tan\alpha\cos\beta)\}$

Q.162 Transform the integral $I = \iiint (x + y + z)xyz dxdydz$ taking over the volume bounded by $x = 0, y = 0, z = 0, x = y + z = 1$, substituting $u = x + y + z, x + y = uv, y = uvw$, and hence evaluate it value.

Q.163 Transform the integral I=$\iiint (x + y + z)^n xyz dxdydz$ taking over the volume bounded by $x = 0, y = 0, z = 0, x + y + z = 1$, substituting $u = x + y + z, x + y = uv, y = uvw$, and hence evaluate its value.

Q.164 If r and r' be the distance of a point in plane of reference from two fixed point at a distance 2c apart on the axis of x then between corresponding limits $\iint 2cy dxdy = \iint rr dr dr$.

Q.165 Transform $\iiint (x - y)(y - z)(z - x) dxdydz$ into one in which u,v,w are the independent variables, where $u^2 = x, y, z \; 1/v = 1/x + 1/y + 1/z, w^2 = x^2 + y^2 + z^2$.

Q.166 Show that $\iiint \frac{dxdydz}{(x+y+z+1)^3} = \frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$ throughout the volume bounded by the coordinate planes and the plane $+y + z = 1$.

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Multiple Integration

Q.167 By using the transformation $x + y = u, y = uv$, show that $\int_0^1 \int_0^{1-x} e^{y/(x+y)}\, dxdy = 1/2(e-1)$.

Q.168 Show that if l, m, n are all positive $\iiint x^{l-1}y^{m-1}z^{n-1}\, dxdydz = \frac{a^l b^m c^n \Gamma(1/2)\Gamma(m/2)\Gamma(n/2)}{\Gamma\left(\frac{l+m+n+2}{2}\right)}$ where the multiple integral is taken through out the part of ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, which lies in the $+ve$ octant.

Q.169 Evaluate $\iiint xyz\, dx\, dy\, dz$ taking throughout the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$.

Q.170 Find the value of $\iint x^{l-1}y^{-1}e^{x+y}\ dxdy$, extended to all positive values, subject to the condition $x + y < h$.

Q.171 Evaluate $\iiint x^{-1/2}y^{-1/2}z^{-1/2}(1-x-y-z)^{1/2}dx\, y\, dz$ extended to all positive values of the variables subject to the condition $x + y + z < 1$.

Q.172 Evaluate $\iiint e^{x+y+z}\, dxdydz$ taken over positive octant such that $x + y + z \leq 1$.

Q.173 Find the value of $\iiint \log(x+y+z)dx\, dy$ the integral extending over all positive values of x, y, z subject to the condition $x + y + z < 1$.

Q.174 Evaluate $\iiint \frac{dx_1 dx_2 \ldots dx_n}{\sqrt{(1-x_1^2-x_2^2 \ldots -x_n^2)}}$ integral being extended to all positive values of the variable for which the expression is real.

Q. 175 Prove that $\iiint \frac{dxdydz}{\sqrt{(a^2-x^2-y^2-z^2)}} = \frac{\pi^2 a^2}{8}$ the integral being extended for all position value of the variable for which the expression is real.

Q.176 Evaluate $\iiint \sqrt{(a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2)}dxdydz$ taken throughout the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Q.177 Prove that $I = \iiint dxdydzdw$, for the all the values of the variable for which $x^2 + y^2 + z^2 + w^2$ is not less then a^2 and not greater then b^2 is $\frac{\pi^2}{32}(b^4 - a^4)$.

Q.178 Evaluate $\iiint \frac{dxdydz}{x+y+z+1}$ over the volume bounded by the co-ordinate planes and the planes $x + y + z = 1$.

Q.179 Find the value of $x^{-2}/a^2 + y^2/b^2 + z^2/c^2 = 1$.

Q.180 Find the volume enclosed by the surfaces $x^2 + y^2 = cz, x^2y^2 = 2ax, z = 0$.

Q.181 Find the volume of surface determined by $z^2 + a^2y^2/x^2 = c^2$, which is contained between the planes $x = 0$ and $x = a$.

Q.182 The axes of two right circular cylinders of the same radius 'a' intersect at the right angles prove that the volume which is inside both the cylinder is $\frac{16}{3}a^3$.

Q.183 Find the volume cut from a sphere of radius a by a right circular cylinder with b as radius of the base and whose axis passes through the center of the sphere.Type equation here.

Q.184 Find the volume bounded by $y^2 + z^2 = 4ax, y^2 = ax, x = 3a$.

Q.185 Find the volume of the cylindrical column standing on the area common to the parabolas $x = y^2, y = x^2$ as base and cut of by the surface$z = 12 + y - x^2$.

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Multiple Integration

Q.186 Through a diameter of the upper base of a right cylinder of altitude ‘a’ and radius ‘r’ pass two planes which touch the lower base on opposite side. Find of the volume of the cylinder included between the two planes.

Q. 187 The curve $z(a^2 + x^2)^{3/2}$=a^4 lying in the $z\,x$ planes revolves about the z-axis. Find the volume in + ve octant included between the surface and plane $x = 0, x = a, y = 0, y = a$.

Q.188 Find the volume bounded by the paraboloid $x^2 + y^2 = 1 + z$ and $z = 0$

Q.189 Find the volume cut from the sphere $x^2 + y^2 + z^2 = a^2$ by the cylinder $x^2 + y^2 = ax$.

Q.160 Find the volume of the potion of the cylinder determined by the equation $x^2 + y^2 - 2ax = 0$, which is the intercepted between the planes $z = x\ \tan\alpha$, $z = x \tan\beta$.

Q.161 Find the volume of the solid cut off by the surface $z = (x + y)^2$ from right prism whose base in the planes $z = 0$ is the rectangle bounded by the lines $x = 0, y = 0, x + y = 1$.

Q.162 The surface enclosed by the planes $x = 0, y = 0, x + y = 1$ and surface $zx = e^{x+y}$ is filled with matter whose density at any point (x, y, z) is given by $\rho = (x/y)^{2/3}$ show that the whole mass is $2\pi\,(e - 1)/\sqrt{(3)}$.

Q.163 Show that the volume common to the surface $y^2 + z^2 = 4ax$ and $x^2 + y^2 4ax$ and $x^2 + y^2 = 2ax$ is $\frac{2}{3}(3\pi + 8)a^2$.

Q.164 A right cone has its vertex in the surface of the sphere and its axis coincident with diameter of the sphere, passing through that point. Find the volume common to the cone and the sphere.

Q.165 A right cone has its vertex at the center and its axis coincident with the diameter passing through that point. Find the volume common to the cone and sphere.

Q.166 The sphere $x^2 + y^2 + z^2 = a^2$ is pierced by the cylinder $(x^2 + y^2)^2 = a^2(x^2 - y^2)$, prove that the volume of the sphere that lies inside the cylinder is $\frac{8}{3}\left(\frac{\pi}{4} + \frac{5}{3} - \frac{4\sqrt{(3)}}{3}\right)a^3$.

Q.167 Find the volume of the solid bounded by the surface $(x^2 + y^2 + z^2)^3 = 27a^3xyz$.

Q.168 Prove of the area of the surface of the paraboloid $az = x^2 + y^2$ which lies between the planes $z = 0, z = a$, is $\frac{\pi}{6}\left[5\sqrt{(5) - 1}\right]a^2$.

Q.169 Find the area of the surface $az = xy$ that lines inside the cylinder $(x^2 + y^2)^2$=$2a^2xy$.

Q.170 Prove that area of the surface of the sphere $x^2 + y^2 + z^2 = 1$ that lies inside the cylinder $2x^2(x^2 + y^2) = 3(x^2 - y^2)$ is $2\pi - 4\sqrt{(2)}\left[3\log\left\{\sqrt{(3) +}\sqrt{(2)}\right\}\right] - 2\log\left[1 + \sqrt{(2)}\right]$.

Q.171 Find the volume bounded by $y^2 = x + 1, y^2 = -x + 1, z = x + 4$.

Q.172 Express the volume contain between the surface whose equation are $x^2 + y^2 + z^2 = a^2, x^2 + y^2 = a^2$, $z^2 = a$ and the co-ordinate planes in the form $V = \iint zdxdy, V = \iint xdzdy$ and determined the volume of V.

Q.173 Find the volume of the wedges cut from the cylinder $4x^2 + y^2 = a^2$ by the planes $z = 0$ and $z = my$.

Q.174 Find the area of the cylinder $x^2 + y^2 = 6y$ lying inside the sphere $x^2 + y^2 + z^2 = 36$.

Q.175 Compute the triple integral of $f(r, \theta, z) = r^2$ over the region R bounded by the paraboloid $r^2 = 9 - z$ and the planes $z = 0$.

Multiple Integration

Q.176 Compute the tripe integral of $F(\rho, \theta, \emptyset) = 1/\rho$ over the region R in the first octant bounded by the cones $\emptyset = \frac{\pi}{4}$ and $\emptyset = tan^{-1}2$ and the sphere $\rho = \sqrt{6}$.

Q.177 Find the volume with In cylinder $R = 4\cos\theta$ bounded above by the sphere $r^2 + z^2 = 16$ and below by the plane $z = 0$.

Q.178 Find the volume cut from the cone $\emptyset = \frac{\pi}{4}$ by the sphere $\rho = 2a\cos\emptyset$.

Q.179 Find the mass of plate in the form of a quadrant of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose density unit area is given by $e = kxy$, $\frac{ka^2b^2}{8}$

Q.180 Find the area lying between the parabola $y = 4x - x^2 4$ the line $y = x$

Q.181 Find the volume under the plane $x + y + z = 6$ & above the xy −planes bounded by the lines $2x = 3y, y = 0, \& \ x = 3$

Q.182 Transform $\iint vdxdy$ to polar coordinate,v being a function of x & y

Q.183 Transform $\int_0^a \int_0^{a-x} vdxdy$ by the substitution $x + y = v, y = vu, v$ being a function of x & y

Q.184 Transform the integral $I = \int_0^{\pi/2} \int_0^{\pi/2} \sqrt{\frac{\sin\emptyset}{\sin\emptyset}} d\emptyset d\theta$ the substitutions $x = \sin\emptyset\cos\theta, y = \sin\emptyset\sin\theta$ & show that its value is

Q.185 Find mass of solid bounded by the coordinate planes & the surface $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r = 1$, when density at any point of the volume is $kx^m y^n z^o$

Q.186 Evaluate the integral $I = \iiint \frac{dxdydz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ for all positive value of the variable for which the integral is real $\frac{a^2\pi^2}{8}$

Q.187 Find by the double integral that the area laying inside the circle $r = a\sin\theta$ & outside the cardioid $r = a(1 - \cos\theta)$ is $a^2(1 - \pi/4)$

Q.188 Find the double integration the area laying inside the coordinate $r = a(1 + \cos\theta)$ & outside the circle $r = a$.

Q.189 Find the volume of cylinder $x^2 - y^2 - ax = o$ bounded by the planes $z = 0$ & $z = x$

Q.190 Find the volume under the plane $z = x + y$ & above the area cut from the first quadrant by the ellipse $4x^2 + 9y^2 = 36$

Q.191 Prove that volume common to the cylinders $x^2 + y^2 = a^2$ & $x^2 + z^2 = a^2$ is $\frac{16}{3}a^3$

Q.192 Find the volume in the positive octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ hence or otherwise, find the volume of ellipsoid

Q.194 Find the triple integration the volume cut off from the cylinder $x^2 + y^2 = ax$ by the planes $z = mx$ & $z = nx$

Q.195 Find the volume of first octant bounded by $z = x^2 + y^2$ & $y = 1 - x^2$

Q.196 Find the volume bounded by the cylinder $x^2 + y^2 = a^2$ & the cone $x^2 + y^2 = z^2$

Q.197 By using the transformation$x + y = u$, y=uv, prove that $\int_0^1 \int_0^{1-x} e^{y/x+y} dxdy = \frac{e-1}{2}$

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Multiple Integration

Q.198 By using the transformation $x + y = u, y = uv$, show that the volume of the double integral $\iint [xy(1 - x - y)]^{1/2} dxdy$ taken over the area of the triangle bounded by the lines $x = 0, y = 0$, $x + y = 1$ is $\frac{2\pi}{105}$

Q.199 Transform the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dxdy$ by changing polar coordinate & hence solve it.

Q.200 By changing to polar coordinates, evaluate $\iint xy(x^2 + y^2)^{1/2} dxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$, supposing $x + 3 > 0$

Q.201 Prove that the area in the positive quadrant, bounded by the curves $y^2 = 4ax, y^2 = 4bx$, $xy = d^2$ is $\frac{1}{3}(d^2 - c^2)\log\left(\frac{b}{a}\right)$.

Q.202 Prove that $\iiint x^{l-1}y^{m-1}z^{n-1}\,dxdydz = \frac{a^l b^m c^n}{pqr}\frac{\sqrt{l/p}\sqrt{m/q}\sqrt{n/r}}{\sqrt{\frac{l}{p}+\frac{m}{q}+\frac{n}{r}+1}}$ where $x, y, z \geq 0$ $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$

Q.203 (i) Find the volume of the solid bounded by the coordinate planes & the surface $\left(\frac{x}{a}\right)^{2x} + \left(\frac{y}{b}\right)^{2x} + \left(\frac{z}{c}\right)^{2x} = 1$, where x is the positive integer.

(ii) Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} + \left(\frac{z}{c}\right)^{2/3} = 1$

Q.204 Evaluate $\iiint xyz\,dxdydz$, where $x, y, z > 0$ & $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ without using the Dirichlet's integrals.

Q.205 Find the value of $\iiint \log(x + y + z)dxdydz$, where x, y, z are all positive & $x + y + z < 1$.

Q.206 $\int_0^{\pi/2} d\theta \int_0^{a\sin\theta} dr \int_0^{(a^2-x^2)/a} rdz$

Q.207 Find by double integral the area between $y = \frac{3x}{x^2+2}$ & $4y = x^2$

Q.208 Evaluate $\iint_R (x^2 + y^2)dxdy$ where R is the region bounded by the straight lines $x + y = 0$, $x - y = 2$, $x + y = 2$, $x - y = 0$

Q.209 Evaluate $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$, where $x > 0, y > 0, z > 0$ & $0 \leq x^2 + y^2 + z^2 \leq a^2$

Q.210 $\iint_R [x - y]dxdy$, for $R = [0,1,0,1]$

Q.211 A function is defined on a rectangle [0,1,0,1] as $f(x,y) = \begin{cases} \frac{1}{2}, when\ y\ is\ rational \\ x, when\ y\ is\ rational \end{cases}$ The iterated integral or repeated integral $\int_0^1 dy \int_0^1 f dx = 1/2$, but other iterated integral does not exist.

Q.212 Evaluate $\iint_R f(x,y)dxdy$ over the rectangle $R = [0,1,0,1]$ where $f(x,y) = \begin{cases} (x+y), x^2 < y < 2x^2 \\ 0, \quad elsewhere \end{cases}$

Q.213 Evaluate $\iint_R [x + y]\,dxdy$, over the rectangle R, [0,1,0,2], where $[(x + y)]$ denotes the greatest integer less than or equal to (x+y)

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Multiple Integration

Q.214 $\iint_R \sqrt{|y-x^2|}dxdy = {3\pi + 8}/_{6}$ where $R = [-1,1,0,2]$

Dirichlet Integrals and Liouville's Theorem

Q.1The triple integral $\iiint_V x^{l-1}y^{m-1}z^{n-1}dxdydz$, where V the denotes the volume of space bounded by the co-ordinate planes $x=0, y=0, z=0$ and the plane $x+y+z=1$, is called Dirichlet's Integral.

Q.2 If V is the region given by $x \geq 0, y \geq 0, z \geq 0$ and $x+y+z \leq 1$, then
$\iiint_V x^{l-1}y^{m-1}z^{n-1}dxdydz = \frac{(l)(m)(n)}{(l+m+n+1)}$

Q.3 Liouville extended Dirichlet's theorem as fallow If x, y, z are all positive such that $h_1 \leq x+y+z \leq h_2$, then

$$\iiint f(x+y+z)\, x^{l-1}y^{m-1}z^{n-1}dxdydz = \frac{(l)(m)(n)}{(l+m+n)}\int_{h_1}^{h_2} f(u)u^{l+m+n-1}\,du$$

Q.4 Evaluate $\iiint x^{l-1}y^{m-1}z^{n-1}\,dxdydz$, where x, y, z are always positive but limited by the condition $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$.

Q.5 Evaluate $\iiint xyz\,dxdydz$ taken from throughout the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \leq 1$.

Q.6 Evaluate $\iiint_R (x+y+z+1)^2\,dxdydz$, where R is the region $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1$.

Q.7 Evaluate the integral $\iiint \frac{dx\,dy\,dz}{\sqrt{(a^2-x^2-y^2-z^2)}}$ the integral being extended for all positive values of the variables for which the expression is real.

Q.8 Evaluate $\iiint \left(\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}\right) dx\,dy\,dz$, integral being taken over all positive values of x, y, z such that $x^2+y^2+z^2 \leq 1$.

Q.9 Evaluate $\iiint \sqrt{(a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2)}dxsydz$ taken throughout the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$.

Q.10 Show that $\iiint x^{l-1}y^{m-1}z^{n-1}dx\,dy\,dz = a^l b^m c^n \frac{(l)(m)(n)}{(l+m+n+1)} h^{l+m+n}$ where the integral is extended over all the non-negative values of x, y, z such that $x/a + y/b + z/c \leq h$.

Q.11 Evaluate the integral $\iiint \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right) xyz\,dx\,dy\,dz$ over the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$, lying in the positive octant.

Q.12 (i) If l, m, n are all positive, show that $\iiint x^{l-1}y^{m-1}z^{n-1}\,dx\,dy\,dz = \frac{a^l b^m c^n.\Gamma\left(\frac{1}{2}l\right)\Gamma\left(\frac{1}{2}m\right)\Gamma\left(\frac{1}{2}n\right)}{8\Gamma\left\{\frac{1}{2}(l+m+n)+1\right\}}$, where the multiple integral is taken throughout out the part of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$, which lies in the positive octant.

(ii) Find the volume of the following ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$

Hint: - Volume $=8\iiint dx\,dy\,dz = 8 \times \frac{1}{6}\pi abc$, where the integral is taken throughout the part of ellipsoid in the positive octant

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Q.13 (i) Show that $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3} = \frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$ through out the volume bounded by the co-ordinate planes and the planes $x + y + z = 1$

(ii) Show that $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3} = \frac{1}{16}\log\left(\frac{256}{e^5}\right)$ taken throughout the tetrahedron by the planes $x = 0, y = 0, z = 0, x + y + z = 1$.

Q.14 Prove that $\int_0^\infty \int_0^\infty f(x+y)x^\alpha y^\beta dxdy = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)} \int_0^\infty f\,(u)u^{\alpha+\beta+1}$ du when extended to all positive values of variables x and y subject the condition $x + y < \infty$.